Midterm 1 - Review - Problems

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1 Linear equations

Problem 1:

Solve the following system (or say it has no solutions):

$$\begin{cases} x + 2y - z = 2 \\ x + 2y - 2z = 0 \\ 2x + 4y - 2z = 1 \end{cases}$$

Problem 2

Use the following LU factorization of A to solve the equation $A\mathbf{x} = \mathbf{b}$:

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

2 Matrix products and inverses

Problem 3

Calculate AB (or say it's undefined), where:

(a) $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Problem 4

Find the inverse of the following matrix:

$$A = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

Problem 5

Does the inverse of the following matrix exist?

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}$$

Hint: This is a one-liner!

3 Linear Transformations

Problem 6

Assume $T : \mathbb{R}^3 \to \mathbb{R}^3$ maps $\mathbf{e_1}$ to $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e_2}$ to $\begin{bmatrix} 3 \\ -1 \\ 0 \end{bmatrix}$ and $\mathbf{e_3}$ to $\begin{bmatrix} 5 \\ 4 \\ 0 \end{bmatrix}$. Find the matrix of T.

Problem 7

Assume $T : \mathbb{R}^2 \to \mathbb{R}^2$ rotates points in the plane by $\frac{3\pi}{2}$ radians. Find the matrix of T.

4 $Nul(A)$, $Col(A)$, Linear dependence, Span

Problem 8

(a) For the following matrix A, find a basis for $Nul(A)$, $Col(A)$.

(b) Are the columns of A linearly independent? Do they span $\mathbb{R}^4$?

$$A = \begin{bmatrix} 3 & -1 & 7 & 3 & 9 \\ -2 & 2 & -2 & 7 & 6 \\ -5 & 9 & 3 & 3 & 4 \\ -2 & 6 & 6 & 3 & 7 \end{bmatrix} \sim \begin{bmatrix} 3 & -1 & 7 & 0 & 6 \\ 0 & 2 & 4 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

5 True/False Extravaganza

Problem 9

(a) If A is a 3×2 matrix, then $A\mathbf{x} = \mathbf{0}$ always has a nontrivial solution.

(b) If $T : \mathbb{R}^n \to \mathbb{R}^n$ is a linear transformation that is also onto, then the equation $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b}$.

(c) If $AB = I$, then A is invertible

(d) If A and B are 2×2 matrices such that $A \neq O$ and $B \neq 0$, then $AB \neq O$ (where O is the zero matrix)

(e) If A is $n \times n$ and has n pivot rows, then the columns of A span $\mathbb{R}^n$

(f) If A is invertible, then $Nul(A) = \{\mathbf{0}\}$

(g) If $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}$ are in $\mathbb{R}^4$ and $\mathbf{v_3} = 2\mathbf{v_1} + \mathbf{v_2}$, then $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly dependent.

(h) If $\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}$ are in $\mathbb{R}^4$ and $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly dependent, then $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is linearly dependent.